Griffiths.

② ~~tt~~, 7.34. $\mathcal{M}_1 \mathcal{M}_2^*$

$$= \frac{g_e^4}{(p_1-p_3)^2(p_1-p_4)^2} [\bar{u}_3 \gamma^\mu u_1][\bar{u}_4 \gamma_\mu u_2][\bar{u}_4 \gamma^\nu u_1]^* [\bar{u}_3 \gamma_\nu u_2]^*$$

~~$= (\cdots) [\bar{u}_2 \gamma_\mu u_2][u_2^* \gamma^0 \gamma^0 \gamma_\nu^\dagger \gamma^0 u_3]$~~

$$= (\cdots) [\bar{u}_3 \gamma^\mu u_1][u_1^* \gamma^0 \gamma^0 \gamma^{\nu\dagger} \gamma^0 u_4][\bar{u}_4 \gamma_\mu u_2][u_2^* \gamma^0 \gamma^0 \gamma_\nu^\dagger \gamma^0 u_3]$$

$$= (\cdots) \bar{u}_3 \gamma^\mu (u_1 \bar{u}_1) \gamma^\nu (u_4 \bar{u}_4) \gamma_\mu (u_2 \bar{u}_2) \gamma_\nu u_3$$

$$= (\cdots) \bar{u}_3 \gamma^\mu \not{p}_1 \gamma^\nu \not{p}_4 \gamma_\mu \not{p}_2 \gamma_\nu u_3$$

$$\boxed{\text{summing over all spin: } (\cdots) \mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_4 \gamma_\mu \not{p}_2 \gamma_\nu \not{p}_3]}$$

$$\mathcal{M}_2 \mathcal{M}_1^* = (\cdots) \bar{u}_4 \gamma^\mu (u_1 \bar{u}_1) \gamma^\nu (u_3 \bar{u}_3) \gamma_\mu (u_2 \bar{u}_2) \gamma_\nu u_4$$

$$= (\cdots) \bar{u}_4 \gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3 \gamma_\mu \not{p}_2 \gamma_\nu u_4$$

$$\boxed{\text{summing over all spin: } (\cdots) \mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3 \gamma_\mu \not{p}_2 \gamma_\nu p_4]}$$

$$\langle |\mathcal{M}_1|^2 \rangle = (\cdots) [\bar{u}_3 \gamma^\mu u_1][\bar{u}_3 \gamma^\nu u_1]^* [\bar{u}_4 \gamma_\mu u_2][\bar{u}_4 \gamma_\nu u_2]^*$$

$$= \boxed{(\cdots) \cdot \mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3] \times \mathrm{Tr}[\gamma_\mu \not{p}_2 \gamma_\nu \not{p}_4]}$$ by Casmir.

Similarly, $\langle |\mathcal{M}_2|^2 \rangle$

$$= (\cdots) [\bar{u}_4 \gamma^\mu u_1][\bar{u}_4 \gamma^\nu u_1]^* [\bar{u}_3 \gamma_\mu u_2][\bar{u}_3 \gamma_\nu u_2]^*$$

$$= \boxed{(\cdots) \mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_4] \times \mathrm{Tr}[\gamma_\mu \not{p}_2 \gamma_\nu \not{p}_3]}$$

We now compute the traces:

$$\mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_4 \gamma_\mu \not{p}_2 \gamma_\nu \not{p}_3]$$

$$= \mathrm{Tr}[\gamma^\mu \gamma^\alpha p_{1\alpha} \gamma^\nu \gamma^\beta p_{4\beta} \gamma_\mu \gamma^\sigma p_{2\sigma} \gamma_\nu \gamma^\lambda p_{3\lambda}].$$

$$= \mathrm{Tr}[\gamma^\mu \gamma^\alpha \gamma^\nu \gamma^\beta \gamma_\mu \gamma^\sigma \gamma_\nu \gamma^\lambda (p_{1\alpha} p_{4\beta} p_{2\sigma} p_{3\lambda})].$$

$$= \mathrm{Tr}\{\gamma^\alpha \gamma^\nu \gamma^\beta (\gamma_\mu \gamma^\sigma \gamma_\nu \gamma^\lambda \gamma^\mu)(p_{1\alpha} p_{4\beta} p_{2\sigma} p_{3\lambda})]$$

$$= \mathrm{Tr}[\gamma^\alpha \gamma^\nu \gamma^\beta (-2\gamma^\lambda \gamma_\nu \gamma^\sigma)(\cdots)]$$

$$= \mathrm{Tr}[-2\gamma^\beta \gamma^\lambda (\gamma_\nu \gamma^\sigma \gamma^\alpha \gamma^\nu)(\cdots)]$$

$$= \mathrm{Tr}[-2\gamma^\beta \gamma^\lambda (4 g^{\sigma\alpha})(\cdots)].$$

$$= \mathrm{Tr}[-8\gamma^\beta \gamma^\lambda g^{\sigma\alpha}\, p_{1\alpha} p_{4\beta} p_{2\sigma} p_{3\lambda}]$$

$$= \mathrm{Tr}[-8\gamma^\beta p_{4\beta} \gamma^\lambda p_{3\lambda}.(p_1 \cdot p_2)]$$

$$= \boxed{\mathrm{Tr}(-8(p_1 \cdot p_2) \not{p}_4 \not{p}_3)}. = -8(p_1 \cdot p_2) 4 (p_4 \cdot p_3).$$

Similarly, $\mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3 \gamma_\mu \not{p}_2 \gamma_\nu \not{p}_4]$

$$= \boxed{\mathrm{Tr}(-8(p_1 \cdot p_2) \not{p}_3 \not{p}_4).} = -8(p_1 \cdot p_2) 4 (p_3 \cdot p_4).$$

$$\mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_3] \times \mathrm{Tr}[\gamma_\mu \not{p}_2 \gamma_\nu \not{p}_4]$$

$$= \mathrm{Tr}[\gamma^\mu \gamma^\alpha p_{1\alpha} \gamma^\nu \gamma^\beta p_{3\beta}] \times (\cdots)$$

$$= \mathrm{Tr}[\gamma^\mu \gamma^\alpha \gamma^\nu \gamma^\beta]\, p_{1\alpha} p_{3\beta}$$

$$= 4(g^{\mu\alpha} g^{\nu\beta} - g^{\mu\nu} g^{\alpha\beta} + g^{\mu\beta} g^{\alpha\nu})\, p_{1\alpha} p_{3\beta} \times (\cdots)$$

$$= 4(p_1^\mu p_3^\nu - g^{\mu\nu}(p_1 \cdot p_3) + p_3^\mu p_1^\nu)$$
$$\times 4(p_{2\mu} p_{4\nu} - g_{\mu\nu}(p_2 \cdot p_4) + p_{4\mu} p_{2\nu}).$$

$$= 16[(p_1 \cdot p_2)(p_3 \cdot p_4) - \cancel{(p_1 \cdot p_3)(p_2 \cdot p_4)} + (p_1 \cdot p_4)(p_3 \cdot p_2)$$
$$\cancel{+ 4(p_1 \cdot p_3)(p_2 \cdot p_4)}$$
$$- \cancel{(p_2 \cdot p_4)(p_1 \cdot p_3)} + 4\cancel{(p_1 \cdot p_3)(p_2 \cdot p_4)} - \cancel{(p_2 \cdot p_4)(p_1 \cdot p_3)}$$
$$+ (p_2 \cdot p_3)(p_1 \cdot p_4) - \cancel{(p_1 \cdot p_3)(p_2 \cdot p_4)} + (p_3 \cdot p_4)(p_1 \cdot p_2)]$$

$$= \boxed{16[2(p_1 \cdot p_2)(p_3 \cdot p_4) + 2(p_1 \cdot p_4)(p_2 \cdot p_3)]}$$

This implies $\mathrm{Tr}[\gamma^\mu \not{p}_1 \gamma^\nu \not{p}_4] \times \mathrm{Tr}[\gamma_\mu \not{p}_2 \gamma_\nu \not{p}_3]$

$$= \boxed{16[2(p_1 \cdot p_2)(p_3 \cdot p_4) + 2(p_1 \cdot p_3)(p_2 \cdot p_4)]}$$

$$\langle |\mathcal{M}|^2 \rangle = \langle |\mathcal{M}_1|^2 \rangle + \langle |\mathcal{M}_2|^2 \rangle - \langle \mathcal{M}_1 \mathcal{M}_2^* \rangle - \langle \mathcal{M}_2 \mathcal{M}_1^* \rangle .$$

$$= \cancel{\left[\frac{g_e^4}{4(p_1-p_3)^4} + \frac{g_e^4}{4(p_1-p_4)^4} \right]}$$

$$= \frac{g_e^4}{4(p_1-p_3)^4}\, 32 \left[(p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) \right]$$

$$+ \frac{g_e^4}{4(p_1-p_4)^4}\, 32 \left[(p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4) \right].$$

$$- \frac{g_e^4}{(p_1-p_3)^2(p_1-p_4)^2} \left[-8 (p_1\cdot p_2)\, 4 (p_3\cdot p_4) \right] \times 2 .$$

$$= \frac{g_e^4}{\cancel{4}} \Bigg[\frac{\cancel{32}^{\,8} \left[(p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) \right]}{(p_1-p_3)^4}$$

$$+ \frac{\cancel{32}^{\,8} \left[(p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4) \right]}{(p_1-p_4)^4}$$

$$+ \frac{\cancel{64}^{\,16} \left[(p_1\cdot p_2)(p_3\cdot p_4) \right]}{(p_1-p_3)^2(p_1-p_4)^2} \Bigg].$$